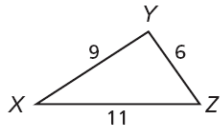
Pre-AP Geometry 5-6 Study Guide: Inequalities in Two Triangles (pp 352-354)

Attendance Problems.

1. Write the angles in order from smallest to largest.

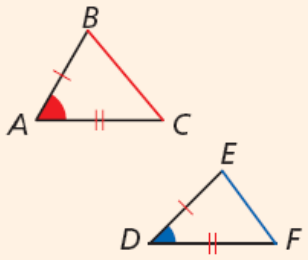
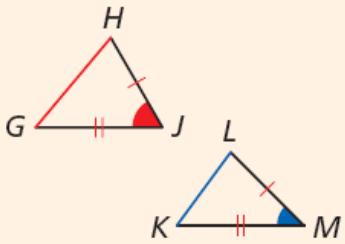


2. The lengths of two sides of a triangle are 12 cm and 9 cm. Find the range of possible lengths for the third side.

I can apply inequalities in two triangles.

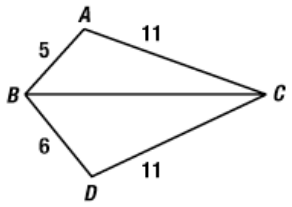
Common Core: CC.9-12.G.CO.10 Prove theorems about triangles.

Theorems Inequalities in Two Triangles

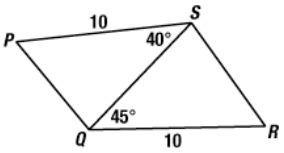
THEOREM	HYPOTHESIS	CONCLUSION
5-6-1 Hinge Theorem If two sides of one triangle are congruent to two sides of another triangle and the included angles are not congruent, then the longer third side is across from the larger included angle.	 $m\angle A > m\angle D$	$BC > EF$
5-6-2 Converse of the Hinge Theorem If two sides of one triangle are congruent to two sides of another triangle and the third sides are not congruent, then the larger included angle is across from the longer third side.	 $GH > KL$	$m\angle J > m\angle M$

Refer to Video example 1 on page 353.

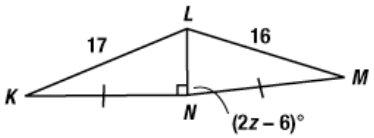
A. Compare $m\measuredangle ACB$ with $m\measuredangle BCD$.



B. Compare PQ and RS.



C. Find the range of values for z.



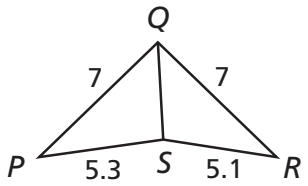
1 Using the Hinge Theorem and Its Converse

A **Compare $m\angle PQS$ and $m\angle RQS$.**

Compare the side lengths in $\triangle PQS$ and $\triangle RQS$.

$PQ = RQ$ $\quad QS = QS$ $\quad PS > RS$

By the Converse of the Hinge Theorem, $m\angle PQS > m\angle RQS$.

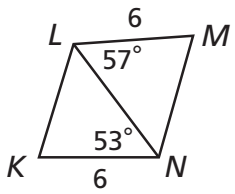


B **Compare KL and MN.**

Compare the sides and angles in $\triangle KLN$ and $\triangle MNL$.

$KN = ML$ $\quad LN = LN$ $\quad m\angle LNK < m\angle NLM$

By the Hinge Theorem, $KL < MN$.

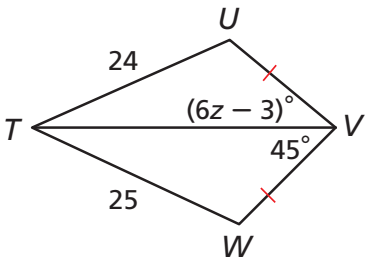


C **Find the range of values for z.**

Step 1 Compare the side lengths in $\triangle TUV$ and $\triangle TWV$.

$TV = TV$ $\quad VU = VW$ $\quad TU < TW$

By the Converse of the Hinge Theorem, $m\angle UVT < m\angle WVT$.



$6z - 3 < 45$ *Substitute the given values.*

$z < 8$ *Add 3 to both sides and divide both sides by 6.*

Step 2 Since $\angle UVT$ is in a triangle, $m\angle UVT > 0°$.

$6z - 3 > 0$ *Substitute the given value.*

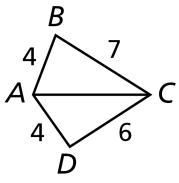
$z > 0.5$ *Add 3 to both sides and divide both sides by 6.*

Step 3 Combine the inequalities.

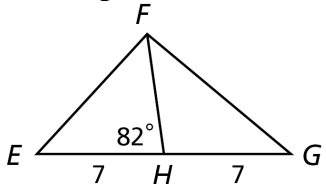
The range of values for z is $0.5 < z < 8$.

Example 1.

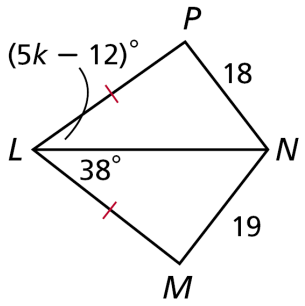
A. Compare $m\measuredangle BAC$ and $m\measuredangle DAC$.



B. Compare *EF* and *FG*.

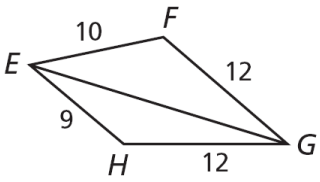


C. Find the range of values for k.

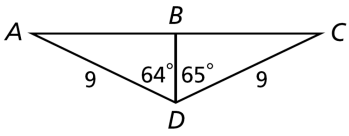


Guided Practice.

3. Compare $m\measuredangle EGH$ & $m\measuredangle EGF$.

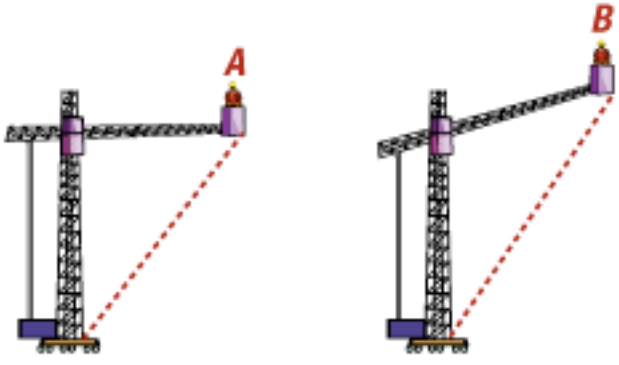


4. Compare *BC* and *AB*.



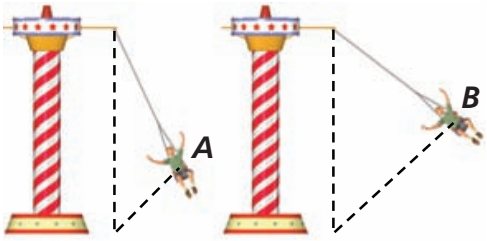
Refer to video example 2.

The boom of a crane makes an angle with the main body of the crane. The angle changes based on the position of the boom. The diagram shows the position of the boom at two different settings. Which worker is further from the base of the crane? Explain.



2 *Entertainment Application*

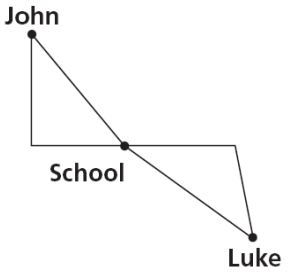
The angle of the swings in a circular swing ride changes with the speed of the ride. The diagram shows the position of one swing at two different speeds. Which rider is farther from the base of the swing tower? Explain.



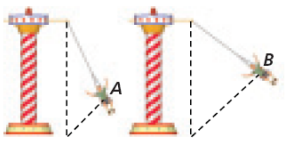
The height of the tower and the length of the cable holding the chair are the same in both triangles.

The angle formed by the swing in position *A* is smaller than the angle formed by the swing in position *B*. So rider *B* is farther from the base of the tower than rider *A* by the Hinge Theorem.

Example 2. John and Luke leave school at the same time. John rides his bike 3 blocks west and then 4 blocks north. Luke rides 4 blocks east and then 3 blocks at a bearing of S 10º E. Who is farther from school? Explain.



5. Guided Practice. When the swing ride is at full speed, the chairs are farthest from the base of the swing tower. What can you conclude about the angles of the swings at full speed versus low speed? Explain.

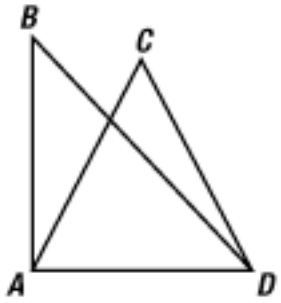


Refer to video example 3.

Write a two-column proof.

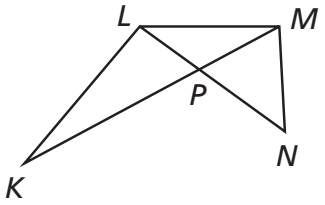
Given: $\overline{AB} \cong \overline{AC}$

Prove: $BD > CD$



3 Proving Triangle Relationships

Write a two-column proof.
Given: $\overline{KL} \cong \overline{NL}$
Prove: $KM > NM$



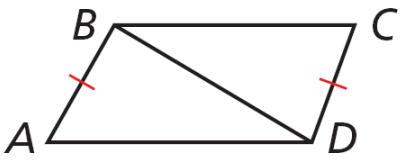
Proof:

Statements	Reasons
1. $\overline{KL} \cong \overline{NL}$	1. Given
2. $\overline{LM} \cong \overline{LM}$	2. Reflex. Prop. of $\cong$
3. $m\angle KLM = m\angle NLM + m\angle KLN$	3. $\angle$ Add. Post.
4. $m\angle KLM > m\angle NLM$	4. Comparison Prop. of Inequal.
5. $KM > NM$	5. Hinge Thm.

Example 3. Write a two-column proof.

Given: $\overline{AB} \cong \overline{CD}, m\measuredangle ABD > m\measuredangle CDB$

Prove: AD > CB



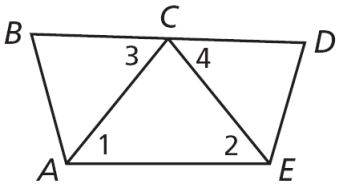
Guided Practice.

C is the midpoint of $\overline{BD}$.

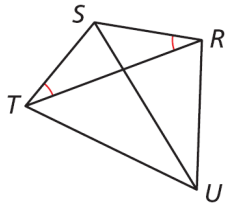
6. Given: $m\measuredangle 1 = m\measuredangle 2$

$m\measuredangle 3 > m\measuredangle 4$

Prove: AB > ED



7. Given: $\measuredangle SRT \cong \measuredangle STR$
$TU > RU$
Prove: $m\measuredangle TSU > m\measuredangle RSU$



5-6 Inequalities in two triangles: (pp 355) 11, 13, 15, 18, 24, 30.